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THE SPECTRUM OF THE $O(g^4)$ SCALE-INVARIANT LIPATOV KERNEL

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Abstract

The scale-invariant $O(g^4)$ Lipatov kernel has been determined by t-channel unitarity. The forward kernel responsible for parton evolution is evaluated and its eigenvalue spectrum determined. In addition to a logarithmic modification of the $O(g^2)$ kernel a distinct new kinematic component appears. This component is infra-red finite without regularization and has the holomorphic factorization property necessary for conformal invariance. It gives a reduction (of $\sim 65\alpha_s^2/\pi^2 \sim 0.15$) in the power growth of parton distributions at small-x.

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1. INTRODUCTION

The BFKL pomeron[1] or, more simply, the Lipatov pomeron, has recently attracted growing attention, both from the theoretical and the experimental side. The BFKL equation resums leading logarithms in $1/x$. When applied in the forward direction, at large Q^2, it becomes an evolution equation for parton distributions. The Lipatov pomeron solution of the equation predicts that a growth of the form

$$F_2(x,Q^2) \ \sim \ x^{1-\alpha_0} \ \sim \ x^{-\frac{1}{2}} \ , \tag{1}$$

where $\alpha_0 - 1$ is the leading eigenvalue of the forward $O(g^2)$ Lipatov kernel, should be observed in the small-x behaviour of structure functions. The BFKL pomeron is important in hard diffractive processes in general, for example deep-inelastic diffraction[2], and, perhaps, in rapidity-gap jet production[3]. BFKL resummation is also anticipated to play a key role in all semi-hard QCD processes [4], where there is a direct coupling of the hard scattering process to the pomeron. It is one of the major results of the HERA experimental program that a growth similar to that of (1) is observed[5].

From both a theoretical and an experimental viewpoint, it is vital to understand how the BFKL equation, and (1) in particular, is affected by next-to-leading logarithm contributions. In recent papers [6, 7] the scale invariant part of the $O(g^4)$ next-to-leading kernel has been determined by reggeon diagram and t-channel unitarity techniques. In this paper we summarise some newly derived properties of this kernel, concentrating on the forward direction relevant for the evolution of parton distributions.

The new kernel is initially expressed in terms of transverse momentum integrals. We have evaluated these integrals explicitly in the forward direction. The results for the connected part of the kernel can be presented in terms of finite combinations of logarithms. We find that there are two components. The first simply has the structure of the $O(g^2)$ kernel but with additional logarithms of all the transverse momenta involved. This component can also be obtained by squaring the $O(g^2)$ kernel. The infra-red divergences it produces after integration are regulated by the disconnected part of the kernel. Also, for this component the new eigenvalues are trivially obtained by squaring the $O(g^2)$ eigenvalues.

The second component is a new kinematic form which appears for the first time at $O(g^4)$. It has a number of important properties. Firstly not only is it separately finite, but it has no singularities generating infra-red divergences after integration. It therefore requires no regulation. A completely new eigenvalue spectrum is produced,

which we give an explicit expression for. We find that the spectrum posesses the fundamental property of holomorphic factorization, which is a necessary condition for conformal symmetry of the kernel[8].

Since the new component appears first at $O(g^4)$ and also has the same conformal invariance property as the leading-order kernel, we anticipate that scale-ambiguities in its absolute evaluation will appear only in higher-orders. That is to say it makes as much sense to evaluate this new component at a fixed value of α_s as it did to evaluate the leading-order contribution with such a value. Consequently we can quote a result for the modification of α_0 by this contribution. There is a reduction of just the right order of magnitude to give an improvement in the phenomenology, while preserving a significant effect.

We are unable, as yet, to give a complete result for how (1) is modified by our results. This is because we must first determine how the scale-invariance of the $O(g^4)$ kernel is broken by the off-shell renormalization scale so that, presumably, $g^2/4\pi \to \alpha_s(Q^2)$. This is non-trivial since we expect that all the transverse momenta in the diagrams of the kernel will be involved in the scale-breaking. Fadin and Lipatov have already calculated[9] the full reggeon trajectory function, that gives the disconnected piece of the kernel, in the next-to-leading log approximation - including renormalization effects. The diagram structure we have anticipated is what is found, but there are additional scale-breaking internal logarithm factors involved. As outlined in [7], we hope to determine the scale-breaking logarithms, that occur in the remainder of the kernel, by an extension of the Ward identity plus infra-red finiteness analysis that gives the scale-invariant kernel.

The contribution of (t-channel) four-particle nonsense states to the connected part of the $O(g^4)$ kernel is given in [6] as a sum of transverse momentum integrals

$$(g^2N)^{-2}K^{(4n)}_{2,2}(k_1,k_2,k_3,k_4)_c \;=\; K_2 \;+\; K_3 \;+ K_4 \;\; . \tag{2}$$

To be consistent with the diagrammatic notation used below, we introduce a momentum conserving δ-function - compared to the definition given in [6] - and write

$$K_i \;=\; (2\pi)^3 \; \delta^2(k_1+k_2-k_3-k_4) \; \tilde{K}_i, \tag{3}$$

with

$$\tilde{K}_2 \;=\; - \sum_{1<->2} \left(\frac{k_1^2 J_1(k_1^2) k_2^2 k_3^2 + k_1^2 J_1(k_1^2) k_2^2 k_4^2 + k_1^2 k_3^2 J_1(k_3^2) k_4^2 + k_1^2 k_3^2 k_4^2 J_1(k_4^2)}{(k_1-k_3)^2} \right), \tag{4}$$

$$\tilde{K}_3 \ = \sum_{1<->2} J_1((k_1-k_3)^2)\Big(k_2^2k_3^2+k_1^2k_4^2\Big), \tag{5}$$

and

$$\tilde{K}_4 \ = \sum_{1<->2} k_1^2k_2^2k_3^2k_4^2 \ I(k_1,k_2,k_3,k_4), \tag{6}$$

where

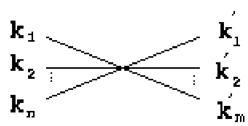
$$J_1(k^2) \ = \ \frac{1}{(2\pi)^3}\int d^2q \frac{1}{q^2(k-q)^2} \tag{7}$$

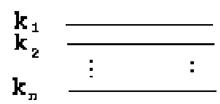
and

$$I(k_1,k_2,k_3,k_4) \ = \ \frac{1}{(2\pi)^3}\int d^2p \frac{1}{p^2(p+k_1)^2(p+k_4)^2(p+k_1-k_3)^2}. \tag{8}$$

The Ward Identity constraint that the kernel should vanish when $k_i \to 0, \ i=1,..,4$, together with infra-red finiteness, determine the relative weights of K_2, K_3 and K_4.

It will be convenient to introduce a diagrammatic notation for transverse momentum integrals. We define





$$= \ \ (2\pi)^3\delta^2(\textstyle\sum k_i \ - \sum k_i')(\sum k_i \ )^2 \qquad\qquad = \ \ (1/2\pi)^{3n}\int d^2k_1...d^2k_n \ / \ k_1^2...k_n^2.$$

As we indicated above, we will define all kernels (and parts of kernels) to include a factor $(2\pi)^3\delta^2(\sum k_i - \sum k_i')$. They are then dimensionless and formally scale-invariant. K_2, K_3 and K_4 can be represented as a sum of diagrams of the form shown in Figs. 1(a), 1(b) and 1(c) respectively.

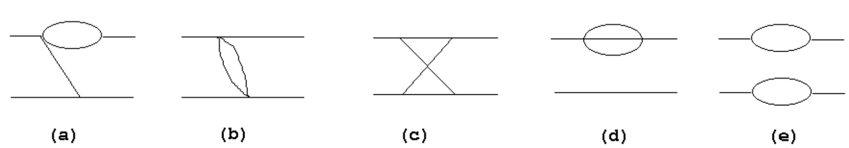


Fig. 1 (a), (b), (c) - connected diagrams for the $O(g^4)$ kernel; (d), (e) - disconnected diagrams.

In [6], the disconnected part of the kernel was assumed to include diagrams of the form of Fig. 1(d) only. Although generated by four-particle nonsense states,

diagrams of the form of Fig. 1(e) were not included. This was essentially because they can not be associated with higher-order reggeization. In fact such diagrams should be included in $K_{2,2}^{(4n)}$. The point being that there is a further contribution to the $O(g^4)$ kernel, from iteration of the two-particle nonsense state, which cancels the contribution of such diagrams. Iteration of the two-particle nonsense state gives a contribution of the form $[K_{2.2}^{(2)}]^2$, which can be represented diagrammatically as in Fig. 2.

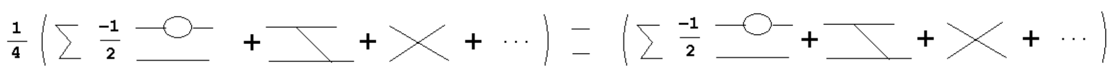


Fig. 2 Iteration of the leading-order kernel via two-particle nonsense states.

For $K_{2,2}^{(4n)}$ to be properly regulated after integration, diagrams of the form of both Fig. 1(d) and Fig. 1(e) must be included and the result is

$$(g^2N)^{-2}K_{2,2}^{(4n)}(k_1,k_2,k_3,k_4)_c \;=\; K_0 \;+\; K_1 \;+\; K_2 \;+\; K_3 \;+K_4\;, \tag{9}$$

where K_0 contains the diagrams of the form of Fig. 1(e). If we use (3), K_0 is given by

$$\tilde{K_0} \;=\; \sum_{1<->2} (2\pi)^3 J_1(k_1^2)J_1(k_2^2\Big(\delta^2(k_2-k_4)+\;\delta^2(k_2-k_3)\Big), \tag{10}$$

K_1 contains the diagrams of the form of Fig. 1(d) and

$$\tilde{K_1} \;=\; -\frac{2}{3}\sum_{1<->2} (2\pi)^3 k_1^2 J_2(k_1^2)k_2^2\Big(k_3^2\delta^2(k_2-k_4)+\;k_4^2\delta^2(k_2-k_3)\Big), \tag{11}$$

with

$$J_2(k^2) \;=\; \frac{1}{(2\pi)^3}\int d^2q\frac{1}{(k-q)^2}J_1(q^2). \tag{12}$$

That the K_0 contributions from $K_{2,2}^{(4n)}$ and $(K_{2,2}^{(2)})^2$ must cancel, determines that the full $O(g^4)$ kernel is given by

$$K_{2,2}^{(4)} \;=\; \frac{1}{2^3}K_{2,2}^{(4n)} \;-\; (K_{2,2}^{(2)})^2 \tag{13}$$

This is the kernel that we wish to evaluate in the "forward" direction $k_1 = -k_2 = k,\ k_3 = -k_4 = k'$. Our result for $K^{(4)}_{2,2}(k,-k,k',-k')$ is a much simpler expression than the full result given by (13).

In writing down(13) we have determined the overall sign by the requirement that the contribution of the four-particle state should be positive. The overall magnitude has been determined by noting that the diagrams of the form of Fig. 1(e) contain only elements that appear in the leading-order kernel and their contribution in $K^{(4n)}_{2,2}$ is unambiguous. This implies that these diagrams should occur in $K^{(4n)}_{2,2}$ (and therefore $(K^{(2)}_{2,2})^2$) with an absolute magnitude that is equal to that obtained by simple-minded iteration of the leading-order kernel.

The major technical problem in determining $K^{(4)}_{2,2}(k,-k,k',-k')$ is the evaluation of the box graph, i.e. $I(k,-k,k',-k')$ defined in (8). As we will show in detail in [10], if we regularize I with a mass term m^2 in each propagator it can be evaluated as a sum of logarithms associated with each of the possible "two-particle" thresholds in the external momenta. As $m^2 \to 0$, we obtain

$$\begin{aligned} 2\pi^2 I[k,k'] &= A_{12} Log[{k'}^2/m^2] + A_{23} Log[k^2/m^2] + A_{34} Log[{k'}^2/m^2] \\ &+ A_{13} Log[(k+k')^2/m^2] + A_{14} Log[k^2/m^2] + A_{24} Log[(k-k')^2/m^2] \end{aligned} \tag{14}$$

where

$$\begin{aligned} A_{12} &= \frac{k^2 - {k'}^2}{k^2(k+k')^2(k-k')^2} & A_{13} &= \frac{1}{k^2 {k'}^2} \\ A_{14} &= \frac{{k'}^2 - k^2}{k'^2(k+k')^2(k-k')^2} & A_{23} &= \frac{{k'}^2 - k^2}{k'^2(k+k')^2(k-k')^2} \\ A_{24} &= \frac{1}{k^2 {k'}^2} & A_{34} &= \frac{k^2 - {k'}^2}{k^2(k+k')^2(k-k')^2} \end{aligned} \tag{15}$$

and so, as $m^2 \to 0$,

$$\begin{aligned} \tilde{K}_4 \ \to\ & \frac{-\,k^2 {k'}^2}{(2\pi^2)} \left(\frac{2({k'}^2 - k^2)}{(k+k')^2(k-k')^2} Log\left[\frac{{k'}^2}{k^2}\right] \right. \\ & \left. + \frac{1}{(k-k')^2} Log\left[\frac{(k-k')^2}{m^2}\right] \ + \ \frac{1}{(k+k')^2} Log\left[\frac{(k+k')^2}{m^2}\right] \right) . \end{aligned} \tag{16}$$

$\tilde{K}_3$ simply gives a contribution of the same form as the last two terms in (16), i.e. as $m^2 \to 0$

$$\tilde{K}_3 \;\to\; \frac{k^2 k'^2}{(2\pi^2)} \left(\frac{1}{(k-k')^2} Log \left[\frac{(k-k')^2}{m^2} \right] + \frac{1}{(k+k')^2} Log \left[\frac{(k+k')^2}{m^2} \right] \right) \qquad (17)$$

Similarly $\tilde{K}_2$ gives

$$\begin{aligned} \tilde{K}_2 \to \frac{-k^2 k'^2}{(2\pi^2)} \Bigg(& \frac{1}{(k-k')^2} (Log \left[\frac{k^2}{m^2} \right] + Log \left[\frac{k'^2}{m^2} \right]) \\ & + \frac{1}{(k+k')^2} (Log \left[\frac{k^2}{m^2} \right] + Log \left[\frac{k'^2}{m^2} \right]) \Bigg). \end{aligned} \qquad (18)$$

The infra-red finiteness of $\tilde{K}_c^{(4n)} \;=\; \tilde{K}_2 + \tilde{K}_3 + \tilde{K}_4$ is now apparent and we can write

$$\begin{aligned} 2\pi^2 \tilde{K}_c^{(4n)} \;&=\; \left(\frac{k^2 k'^2}{(k-k')^2} Log \left[\frac{(k-k')^4}{k^2 k'^2} \right] \;+\; \frac{k^2 k'^2}{(k+k')^2} Log \left[\frac{(k+k')^4}{k^2 k'^2} \right] \right) \\ &\quad - \; \left(\frac{2k^2 k'^2 (k^2 - k'^2)}{(k-k')^2 (k+k')^2} Log \left[\frac{k^2}{k'^2} \right] \right) \\ &= \; \Big(\; \mathcal{K}_1 \; \Big) \; - \; \Big(\; \mathcal{K}_2 \; \Big). \end{aligned} \qquad (19)$$

Note that only $\mathcal{K}_1$ gives infra-red divergences (at $k' = \pm k$) when integrated over k'. These divergences are cancelled by $\tilde{K}_0$ and $\tilde{K}_1$. We will not discuss this cancellation explicitly, but implicitly include $\tilde{K}_0$ and $\tilde{K}_1$ in $\mathcal{K}_1$ for the rest of our discussion.

Apart from the logarithmic factors, $\mathcal{K}_1$ has the same structure as the forward (connected) $O(g^2)$ kernel. Indeed, if we evaluate all of the diagrams generated by Fig. 2 that survive in the forward direction, it is straightforward to show that

$$\mathcal{K}_1 \;=\; (2\pi)^2 (\widetilde{K^{(2)}_{2,2}})^2 \;, \qquad (20)$$

implying from (13) and (19) that

$$\tilde{K}^{(4)}_{2,2} \;\;=\;\; -\,\frac{1}{2^4 \pi^2} (3\mathcal{K}_1 \;+\; \mathcal{K}_2) \;. \qquad (21)$$

There are a number of reasons to believe that the contribution of $(K^{(2)}_{2,2})^2$ to the $O(g^4)$ kernel should produce the scale-dependence of the $O(g^2)$ kernel. Indeed one might be tempted to directly interpret the logarithms in $\mathcal{K}_1$ as associated with coupling constant renormalization in the leading-order kernel. While this can not be simply correct (real ulta-violet renormalization has to be involved to bring in the correct asymptotic freedom coefficients) there may well be some sense in which this is the case. We clearly have to carry out a full scale-breaking analysis to determine what this sense may be.

The interesting part of (21) is the $\mathcal{K}_2$ component. This is finite at $k = \pm k'$, and so does not generate any divergences when integrated. The symmetry properties also determine that this term can only appear at the first logarithmic level (since the antisymmetry of $Log[k^2/k'^2]$ compensates for the antisymmetry of $(k^2 - k'^2)$). It is therefore a completely new feature of the $O(g^4)$ kernel.

We now move on to the eigenvalues of $K^{(4)}_{2,2}$. We use as a complete set of orthogonal eigenfunctions

$$\phi_{\mu,n}(k') \;=\; (k'^2)^\mu \; e^{in\theta} \qquad\qquad \mu = \frac{1}{2} + i\nu, \quad n = 0, \pm 1, \pm 2, ... \tag{22}$$

(Our definition of the kernel requires that we keep a factor of k'^{-2} in the measure of the completeness relation for eigenfunctions relative to [1]). The eigenvalues of $(K^{(2)}_{2,2})^2$ are trivially given by the square of the $O(g^2)$ eigenvalues, and so the essential problem is to determine the eigenvalues of $\mathcal{K}_2$. As a preliminary we first define $\mathcal{K}_2$ for non-integer dimensions.

Since each logarithm in $\mathcal{K}_2$ originates from an integral of the form of J_1 we can replace it by a simple integral of the form

$$\frac{k^2}{2\pi} \int \frac{d^D q}{q^2 (k-q)^2} \;=\; \eta [k^2]^{D/2\ -1} \;, \qquad \eta = \frac{\Gamma[2 - D/2] \Gamma[D/2 - 1]^2}{\Gamma[D-2]}, \tag{23}$$

where $\eta \to 2(D-2)^{-1}$ when $D \to 2$. This gives

$$\mathcal{K}_2 \;=\; 2\eta \, \frac{k^2 k'^2 (k^2 - k'^2)}{(k+k')^2 (k-k')^2} \Big((k^2)^{D/2\ -1} - (k'^2)^{D/2\ -1} \Big) \,. \tag{24}$$

We now write

$$\mathcal{K}_2 \otimes \phi_{\mu,n} \;=\; \mathcal{K}_2^1 \otimes \phi_{\mu,n} - \mathcal{K}_2^2 \otimes \phi_{\mu,n}$$

$$\begin{aligned} &= \lambda_1(\mu,n)\phi_{\mu,n} - \lambda_2(\mu,n)\phi_{\mu,n} \\ &= \lambda(\mu,n)\phi_{\mu,n}, \end{aligned} \tag{25}$$

where

$$\mathcal{K}_2^1 \otimes \phi_{\mu,n} = 2\eta \int \frac{d^D k'}{(k'^2)^2} \frac{(k^2)^{D/2} k'^2 (k^2 - k'^2)\phi_{\mu,n}(k')}{(k-k')^2(k+k')^2}, \tag{26}$$

and

$$\mathcal{K}_2^2 \otimes \phi_{\mu,n} = 2\eta \int \frac{d^D k'}{(k'^2)^2} \frac{k^2 (k'^2)^{D/2} (k^2 - k'^2)\phi_{\mu,n}(k')}{(k-k')^2(k+k')^2}. \tag{27}$$

We take the eigenfunction $\phi_{\mu,n}$ to be defined on a D-dimensional angular space parameterized by $(\theta_1, \theta_2, ..., \theta_{D-1})$ by assuming that $\theta \equiv \theta_{D-1}$. If we define $cos\chi = k \cdot \hat{x}$ and $cos\theta = k' \cdot \hat{x}$, where $\hat{x}$ is an arbitrarily chosen unit vector, the only non-trivial angular integral is

$$\begin{aligned} I_\chi[n] &= \int_0^{2\pi} d\theta \frac{e^{in\theta}}{1 - z(k,k') sin^2\ (\theta - \chi)} \qquad\qquad z[k,k'] = -\frac{4k^2k'^2}{(k^2-k'^2)^2} \\ &= 2\pi e^{in\chi} \left(\frac{k^2-k'^2}{k^2+k'^2}\right) \left[\left(\frac{k'}{k}\right)^n \Theta[k-k'] - \left(\frac{k}{k'}\right)^n \Theta[k'-k]\right]. \end{aligned} \tag{28}$$

if n is an even integer (≥ 0). $I_\chi[n]$ vanishes if n is an odd integer and $I_\chi[-n] = I_\chi[|n|]$.

$I_\chi[n]$ is symmetric under the exchange of k and k', and also is invariant under $k \to 1/k, k' \to 1/k'$. This last invariance is sufficient to show from (26) and (27) that

$$\lambda(\mu\ , n) = \lambda(1-\mu\ , n) \tag{29}$$

Using (28) we obtain from (26) and (27) that, as $D \to 2$,

$$\lambda_1(\mu,n) \to 2\eta \frac{\pi^{D/2}}{\Gamma[D/2]} \Big(\beta(|n|/2 + D/2 + \mu - 1) - \beta(|n|/2 - D/2 - \mu + 2)\Big), \tag{30}$$

and

$$\lambda_2(\mu,n) \to 2\eta \frac{\pi^{D/2}}{\Gamma[D/2]} \Big(\beta(|n|/2 + D + \mu - 2) - \beta(|n|/2 - D - \mu + 3)\Big), \tag{31}$$

where $\beta(x)$ is the incomplete beta function, i.e.

$$\begin{aligned} \beta(x) &= \int_0^1 dy\ y^{x-1}[1+y]^{-1} \\ &= \frac{1}{2}\Big(\psi(\frac{x+1}{2}) - \psi(\frac{x}{2})\Big), \quad \psi(x) = \frac{d}{d\ x} log\Gamma[x]\ . \end{aligned} \tag{32}$$

$\lambda_1(\mu, n)$ and $\lambda_2(\mu, n)$ are separately singular at $D = 2$, but $\lambda(\mu, n)$ is finite, and writing $\Lambda(\nu, n) \equiv \lambda(\frac{1}{2} + i\nu \ , n)$, we obtain

$$\Lambda(\nu, n) = - 2\pi\Big(\beta'(\frac{|n|+1}{2} + i\nu) + \beta'(\frac{|n|+1}{2} - i\nu)\Big). \tag{33}$$

We comment first on the general properties of (33). The symmetry property (29) is clearly reflected in the presence of the two terms. The two terms also give directly the property of holomorphic factorization[8] necessary for conformal symmetry. That is $\Lambda(\nu, n)$ is a sum of two terms, one depending on $(i\nu + 1/2 + n/2)$ and the other on $(i\nu + 1/2 - n/2)$. These two combinations determine respectively the eigenvalues of the holomorphic and anti-holomorphic Casimir operators of linear conformal transformations. Since

$$\beta'(x) = \frac{1}{4}\Big(\psi'(\frac{x+1}{2}) - \psi'(\frac{x}{2})\Big), \tag{34}$$

and

$$\psi'(x) = \sum_{n=0}^{\infty} \frac{1}{(n+x)^2}, \tag{35}$$

$\beta'(x)$ is a real analytic function and it follows from (33) that the eigenvalues $\Lambda(\nu, n)$ are all real.

Note that since the eigenvalues of $K^{(2)}_{2,2}$ can be written as a sum of holomorphic and antiholomorphic components it is clear that the eigenvalues of $(K^{(2)}_{2,2})^2$ can not be. Therefore this part of the $O(g^4)$ kernel is not conformally invariant. This is one of the arguments, referred to earlier, that this term is inter-related with the scale dependendence of the $O(g^2)$ kernel.

Moving on to the modification of α_0, we note that to obtain the contribution to the eigenvalue of $\tilde{K}^{(4)}_{2,2}$ we multiply $\Lambda(\nu, n)$ by $-1/\ 2^4\pi^2$. To compare with $\alpha_0 - 1$ we have to multiply, in addition, by $N^2 g^4/\ (2\pi)^3$, where $N = 3$ for QCD. As we discussed above, since $\mathcal{K}_2$ represents a new kinematic form at $O(g^4)$ we do not expect it to mix with renormalization effects and so it should be legitimate to compare its contribution with $\alpha_0 - 1$ by setting $\alpha_s = g^2/4\pi$. It follows from the above that the leading eigenvalue is $\Lambda(0, 0)$, as it is for the $O(g^2)$ kernel. From (33)-(35) we obtain

the contribution to $\alpha_0 - 1$ from the $\mathcal{K}_2$ term in (21) as

$$
\begin{aligned}
-\ \frac{9\alpha_s^2}{2\pi^3}\Lambda(0,0)\ &=\ \frac{18\alpha_s^2}{\pi^2}\beta'(1/2) =\ -\ \frac{9\alpha_s^2}{2\pi^2}\Big(\ \sum_{n=0}^{\infty}\frac{1}{(n+\ 1/4)^2}\ -\ \sum_{n=0}^{\infty}\frac{1}{(n+\ 3/4)^2}\Big)\\
&=\ -\ \frac{9\alpha_s^2}{2\pi^2}\Big(\ 16\ +\ \frac{16}{25}\ +\ \frac{16}{81}\ +\ ...\ -\ \frac{16}{9}\ -\ \frac{16}{49}\ +\ ...\Big)\\
&\sim\ -\ \frac{9\alpha_s^2}{2\pi^2}\times 14.5\ \sim\ -\ 65\ \frac{\alpha_s^2}{\pi^2}\ \sim\ -\ 0.15
\end{aligned}
\tag{36}
$$

The corresponding contribution from the $\mathcal{K}_1$ term in (21) would be

$$
-\frac{3}{4}\Big(\frac{12}{\pi}Log[2]\ \alpha_s\Big)^2\ \sim\ -\ 0.18 \tag{37}
$$

However, since we have no understanding how the logarithms in this term mix with the renormalization of α_s, this could well be essentially accounted for by the choice of scale in the $O(\alpha_s)$ term. Therefore we believe no attention should be paid to this last number.

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